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Modeling and Analysis of a Manufacturing System with Deadlocks to Generate the Reachability Tree using Petri Net System

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Abstract

Petri Nets have come to play an important role in modeling and analysis. In day manufacturing systems are very complex and need in depth analysis before implementing them. Various models have been proposed for the evaluation of these systems. Stochastic Petri nets are an emerging modeling tool for performance evaluation of manufacturing systems. Petri net based models are executable thus the performance measures can be obtained by direct simulation of the net the AGV system considered is for a job shop consisting of machines with an input and output buffer. The simulation module computes the probability p(i,j) of having j tokens in place p(i). The aim is to find out the minimum number of parts required to keep some level of throughput and to ensure allow completion time for the batch size considered. Deadlock is to be used clearing of machines or AGVs or buffers, and then restart of the system from an initial condition that is known to produce deadlock-free operation under nominal production conditions. The loss of production and the labor cost is reorganizing the system by proper design. In this paper the authors have developed a model for a manufacturing system with deadlock and analyzed it to generate the reachability tree using Petri net system. In this problem the system consists AGV, Machines and L/D station.

Keywords: Petri Net, Firing rate, Modeling, Reachability tree, AGV, Machines, L/D station

Nomenclature

P Place
T Transitions
t firing rate
Greek symbols
μ tangible marking

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1. Introduction

Petri Nets are an effective modeling tool for the description and analysis of concurrency and synchronization in systems, which exhibit cooperative action of different entities. As a graphical tool, they can be used as visual communication aids. As a mathematical tool, it is possible to set up state equation, algebraic equations and other mathematical models governing the behavior of the system. Petri Nets are a tool for study of system Petri net graph models the static properties of system, as a flow chart represents the static properties of computer program [1]. Petri Nets are a tool for study of system Petri net graph models the static properties of system, as a flow chart represents the static properties of computer program [1]. Petri Nets are an effective modeling tool for the description and analysis of concurrency and synchronization in systems, which exhibit cooperative action of different entities. As a graphical tool, they can be used as visual communication aids. As a mathematical tool, it is possible to set up state equation, algebraic equations and other mathematical models governing the behavior of the system. They can be used both by practitioner and theoreticians hence provide a powerful medium of communication between them. Hence methodical and highly realistic models can be developed. In the Bibliography of Petri nets classified according to the application areas. Viswanatham and Narahari [2] describe the use generalized stochastic Petri nets (GSPNs) in the performance studies of automated manufacturing systems. Review of paper Tadeo Murata [3] also gives the properties, analysis and applications of Petri nets. Different areas of application of Petri nets were given in this paper. Examples for modeling Petri nets, extended, subclasses of Petri nets and introduction to timed Petri nets were provided. The concept of time is not given in the original Petri nets. For performance evolution and scheduling problems it is useful to introduce time delays associated with transitions or places in the Petri net models. This Petri net model is called a timed Petri net if the delays are deterministic and if the delays are probabilistic it is called stochastic Petri nets (SPN). This paper contains bibliography of Petri nets classified according to the application areas. Duggan and Browne [4] described a simple Petri net model of a machine performing operation in their production acting control simulation model. Viswanadham and Narahari [5] pointed out the necessity of introducing time in Petri nets. Classical Petri nets are useful in investigating qualitative properties such as mutual exclusion, existence and absence of deadlocks, boudedness and fairness. Viswanadham et al [6] presented comparison of various models used for modeling manufacturing systems. The underlying stochastic process is a Markov process in the case of queuing networks or stochastic Petri nets. An FMS example is provided to compare different modeling approaches. Marsan et al [7] proposed a computationally efficient solution method to compute the transition probability matrix (TPM) of reduced embedded Markov chain. TPM of REMC contains information about tangible states only. They provided some examples of multiprocessor systems performance analysis using GSPNs. Archetti and sciomachen [8] gave some basic methodologies to understand, develop and analyze Petri net-based models of manufacturing systems. It's also showed the application of Petri net modeling and analysis techniques to AGV systems. They point out that the Petri net based models are executable thus the performance measures can be obtained by direct simulation of the net the AGV system considered is for a job shop consisting of three machines with an input and output buffer. Carlier et al [9] modeled a scheduling problem consisting of tasks, resources and constraints using timed Petri nets. In a manufacturing system, scheduling is a typical combinatorial optimization problem refer in Y.W. Kim et al.[10]. The superiority of this approach over others such as network, fault tree and Markov analysis are outlined by G. Thangamani [11]. An integrated Operational Petri Net with Resources model of any Production system is presented by Kumavat and Purohit [12], where a place denotes the different part types and resources, and transitions denote the starting and finishing of an operation through different part types and resources. Balbo et al [13] describe the use of generalized stochastic Petri nets (GSPNs) for the performance evaluation of FMS

1.1. Structure of Petri net system

A Petri Nets is composed of a set of place P, a set of transitions T, an input function I and output function O.

Petri Net	$c = (P, T, I, O)$	
Example:	$P = \{P_1, P_2, P_3, P_4, P_5\}$	
	$T = \{t_1, t_2, t_3, t_4, t_5\}$	
	$I\ (t1) = \{P_1\}$	$O\ (t1) = \{P_2, P_3\}$
	$I\ (t2) = \{P_2\}$	$O\ (t2) = \{P_4\}$
	$I\ (t3) = \{P_3\}$	$O\ (t3) = \{P_5\}$
	$I\ (t4) = \{P_4\}$	$O\ (t4) = \{P_2\}$
	$I\ (t4) = \{P_4, P_5\}$	$O\ (t4) = \{P_1\}$

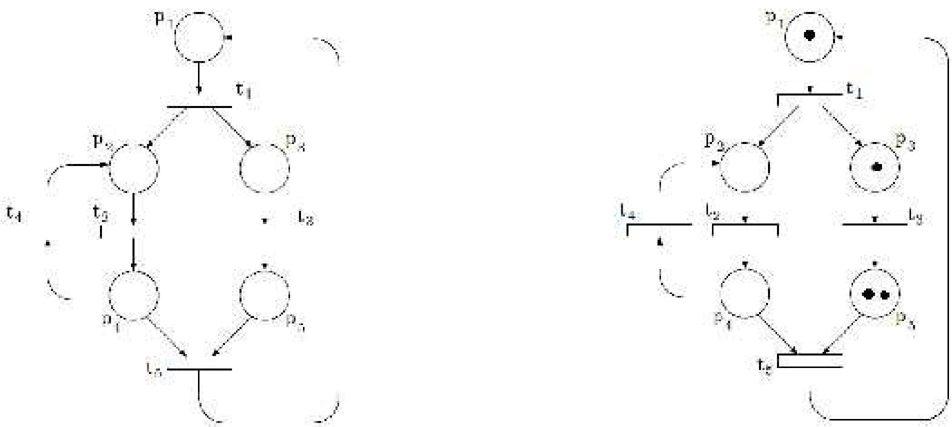
1.2 Graphical Representation

The graphical representation of an example Petri Net is given below in Figure 1(a). A Petri Net graph is a representation of a Petri Net structure. A ‘o’ represents a place and a ‘1’ represents a transition. Directed arcs connected the places and transitions. A Petri Net is directed multi graph as it allows multiple arcs, which are directed.

Marking is an assignment of tokens to the places of Petri Net. The number and position of tokens can change during the execution of a Petri Net. The number and position of tokens can change during the execution of a Petri Net the tokens are used to define the execution of Petri Net the marking μ is defined as an n-vector $\mu = (\mu_1, \mu_2 \ldots \mu_n)$. The number of tokens in place pi is μi=1…n. a marked Petri Net is m= (P, T, I, O, μ). The tokens are represented by small dots ‘.’ In the place of Petri Net .the number of tokens assigned to a place is unbounded.

1.3 Execution Rules (Firing Rule)

A Petri net executes by firing transitions. Whenever a transition is fired, then the tokens from its input places are removed and now tokens are created in its output places to afire a particular transition, it must be enabled, i.e., all its input places must have at least one token per are as an example if a transition t2, has one p1, three p2, and one p3 as its input places then at least there should be 1,3,1 tokens in places p1, p2, and p3 respectively to enable the transition t2, to fire after firing this tokens from input places are removed and new tokens are placed in the output places of the transition .firing a transition changes the marking μ of the Petri net to a new marking. If the transition t_j is fired with a marking μ then the now marking μ’ is denoted as μ‘=δ (μ, tj) if more than one transition is enabled then different marking will result in firing different transition.



(a) (b)

Figure 1. (a) Petri Net Structure (b) Petri Net Structure with initial Marking

In Figure 1(b). $\mu^{\circ} = (1, 0, 1, 0, 2)$

Transition t1 and t3 are enabled new with this present marking, selecting t1 and by firing t1, we obtained fig 3. New marking is obtained by removing tokens from input places of t1, i.e. p1 and putting new tokens into the output places of t1, i.e. p_2, p_3 and p_5.

$\mu = \delta(\mu^{\circ}, t_1) = (0, 1, 2, 0, 1)$

Now t_3 and t_3 are enabled.

By firing t_2 will get $\mu^2 = \delta(\mu^1, t_2) = (0, 0, 2, 1, 1)$

2. Reachability tree generation – basic issues

The steps involved in generating Reachability tree are explained in ‘Petri net Theory and the Modeling of the Systems’ [1]. Viswanatham and Narahri [4] defined Reachability tree with initial marking Mo as the set of all markings reachable from Mo by firing one or more transitions. The steps involved in generating Reachability tree for an inhibitor arc generalized stochastic Petri net (GSPN) model are as follows.

1. The GSPN model of the system is prepared
2. Initial marking of the model is denoted by Mo
3. The set of enabled transitions for this marking are computed by checking the input place to various transitions.
4. Using GSPN firing rules, one of the enabled transitions will be fired. It is given a separate number thus forming a branch. From this marking there may be one or more transitions enabled, further forming branches
5. Using step 4 all the possible enabled transitions will be fired forming several branches.

6. Every time a new marking is generated, it is compared with existing markings. If such a marking already exists, then no further branching takes place for that new is marking.
7. Repeating steps 3-6 gives a tree like structure giving many markings at different levels.
8. This reachability tree consists of vanishing tangible and deadlock, if any, markings.

3. Generalized Stochastic Petri Nets

Marsan et al [7] proposed generalized stochastic Petri nets (GSPNs), an extension of SPNs. In SPN all are timed transitions where as in a GSPN there are two types of transitions.

(i) Immediate transitions which fire in zero time once they are enabled.
(ii) Exponential transitions, which take certain, time to fire.

The use of GSPNs compared to SPNs reduces the solution complexity since

(1) In an SPN every transition is timed and the number of states is equal to the total number of markings in the reachability tree where as in a GSPN, the numbers of states are less.
(2) Some short duration activities can be modeled only from logical point of view in GSPNs

A GSPN is an eight –tuple (P, T, IN, OUT, INH, MO, F, S) where

1. (P, T, IN, OUT, INH, MO) is an inhibitor marked Petri net.
2. T is partitioned into two sets: of immediate transitions and of exponential transitions.
3. F is a firing function that associates to each transition in the set of exponential transitions, an exponential random variable with rate F (M, t). M is a marking in the reachability set of Mo.
4. S is a set of elements called random switches, which associate probability distributions to subsets of conflicting immediate transitions.

A horizontal or vertical line represents an immediate transition and rectangular bar represents an exponential transition. The firing rules for a GSPN are:

(i) If Ti, the set of enabled transitions in the marking Mi, consists of only exponential transition tj fired with probability

$$\frac{F(M_i, t_j)}{\sum_{t_k \in T_i} F(M_i, t_k)} \tag{1}$$

(ii) If Ti comprises one immediate and remaining exponential transition, then the immediate transition is the one that fires.
(iii) If more than two immediate transitions are enabled, then the firing transition will be selected according to a predefined probability distribution, called a random switch.

The marking in a GSPN in which only exponential transitions are enabled Called as tangible markings. The rest of the marking is called vanishing markings. Vanishing markings indicate logical changes in the system. The steps involved in GSPN based performance evaluation are

1. Modeling the system by a GSPN.
2. Generating the marking process i.e. reachability tree.
3. Computing the steady state probability distribution of the marking process using Markovian techniques.
4. Obtaining the required performance measures from the steady state probabilities.

Classical Petri nets are useful in investigating qualitative properties such as mutual exclusion, existence and absence of deadlocks, boudedness and fairness. However, for quantitative performance evaluation, time has to be incorporated in Petri nets. A stochastic Petri net (SPN) is a six tuple (P, T, IN, OUT, MO, F) where (P, T, IN, OUT, MO) is a Petri net and F is a function which associates with each transition in each reachable marking, a random variable. F is known as the firing function. In an SPN when t is enabled in M, the tokens remain in the input places of t and deposited in the output places of t and deposited in the output places of t. under appropriate distributional assumptions, the marking process of SPNs is equivalent to a Markov or semi Markov process with discrete state space. They presented some examples explaining how to evaluate the GSPNs. The steps involved are as follows.

1. The reachability set of GSPN is determined.
2. The embedded Markov chain (EMC) of the GSPN will have same states as that of reachability set. The transition probability matrix (TPM) of the EMC is computed. This TPM contains both vanishing states and tangible states information.
3. As the system stays in the vanishing states only for zero time this information is not required in TPM. A reduced embedded Markov chain (REMC), which contains only tangible states, is derived from EMC. The TPM of REMC is computed using the technique developed by Marsan et al (13). Let p denote this TPM.

4. If y= (y_1, y2…yt) are the stationary probability distribution of the REMC then solving Y P=Y and Σy_i =1 gives the values of all y_i's.
5. If $q_1, q_2 \ldots q_t$ are the steady state probabilities of the tangible states of the marking of the GSPN and m_i be the sojourn time of i th tangible state then the following gives the steady state probabilities of the tangible states.

$$q_i = \frac{y_i . m_i}{\sum_{j=0}^{t} y_j . m_j} \tag{2}$$

6. Once these probabilities are obtained then various performance measures can be computed by interpreting the meaning of tangible markings

Viswandham et al [5] described how GSPN modeling could be used to detect and avoid or prevent deadlocks in manufacturing systems. These can be detected beforehand using GSPN modeling and the performance in the presence of dead locks can be evaluated using Markov techniques for absorbing states. The authors gave methods to prevent and to avoid dead lock in this paper.
Viswanadham et al [3] presented comparison of various models used for modeling manufacturing systems. The underlying stochastic process is a Markov process in the case of queuing networks or stochastic Petri nets. An FMS example is provided to compare different modeling approaches. Silva [14] gives definitions for some terms used in manufacturing.

4. A Manufacturing System with Deadlocks

The systems consist of one load/unload unit, one AGV and one machine. The AGV carried a raw part to and from L/D station to machine and loads it on machine. There is possibility of deadlock in this case. The AGV carried a raw part to the machine and loads it. The machine starts processing the part. Mean while the AGV returns to the L/U station and picks up another raw part and carries it to the machine. Finding the machine completes processing the part it waits for the AGV to unload the part. Once the machine completes processing the part it waits for the AGV to unload the part. Thus both AGV and machine wait for each other indefinitely. This is a simple deadlock situation. Fig 4 and 5 depicts the GSPN model and the reachability tree for the problem considered.

Interpretation of places and transitions

P1	:	AGV available
P2	:	raw parts available
P3	:	AGV ready to carry a raw part
P4	:	AGV ready to carry a finished part
P5	:	AGV carrying a raw part to machine
P6	:	AGV with raw part waiting for the machine
P7	:	machine idle
P8	:	machine processing job AGV released
P9	:	machine waiting for AGV after processing
P10	:	AGV unloading the finished part
P11	:	machine processing job AGV not released
P12	:	AGV unloading the finished part, not released during processing
t_1	:	AGV assigned to raw part
t_2	:	AGV assigned to finished part
t_3	:	AGV starts transporting a raw part
t_4	:	AGV finishes transporting a raw part
t_5	:	AGV released after loading the part
t_6	:	AGV not released after loading the part
t_7	:	machine finished processing a part AGV released
t_8	:	AGV starts unloading a finished part
t_9	:	AGV finishes unloading a finished part
t_{10}	:	AGV not released, carrying a finished part
Firing rate of t4	=	a per hour
Firing rate of t7	=	b per hour
Firing rate of t9	=	c per hour
Firing rate of t10	=	d per hour

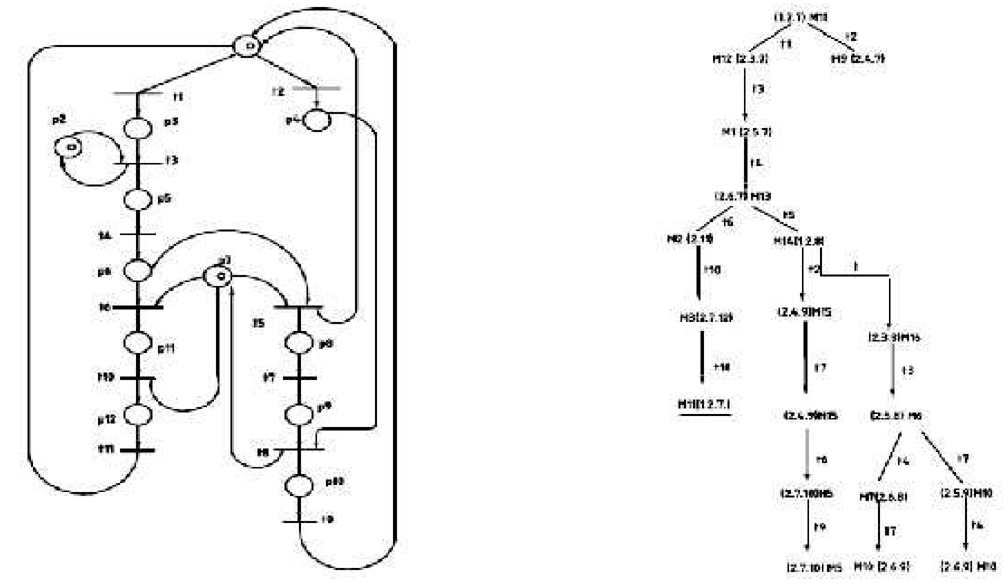
Firing rate of t11 = e per hour

q_1 probability that AGV is assigned to carry raw part

q_2 probability that AGV is assigned to carry finished part

q_3 probability that AGV is released while machine is processing a part

q_4 probability that AGV is not released while machine is processing a part



(a) (b)

Figure 2. (a) GSPN Model for manufacturing system with Deadlocks (b) Reachability tree for manufacturing system with Deadlock

5. EQUIVALENCE TO MARKOV CHAINS

The marking process of a GSPN is a semi markov process with discrete states space and given by the Reachability set. The embedded markov chain (EMC) of this process contains all markings of the Reachability tree Reachability set. The Reachability set. The Reachability set contains three types' markings. Tangible marking are those in which only the time transitions are enabled. In vanishing markings, immediate transitions are enabled. Marking in which no transitions is enabled is known as deadlock marking.

The transition probability matrix (TPM) of the EMC can be calculated using the firing rates of the timed transition. The ij th in the entry TPM denotes the probability i of going from states i to states j. whenever vanishing marking reached, the transition fires in zero time and the sojourn time in vanishing marking zero. Thus performance evaluation, it sufficient to study the tangible marking only. A reduced embedded Markov chain (REMC) consists only tangible marking and the transition probability calculated for this REMC as given by Marsan et al [7].

Let K_t be the total tangible markings and the total number of vanishing markings of GSPN model. The TPM of the EMC is of $(k_t + k_V)$ x $(k_t + k_V)$ dimension. Defining F as a matrix with dimensions k_t x k_V which contains transition probabilities among tangible states, E with dimensions $k_t + k_{V,}$ markings, D contain transition probabilities among vanishing states. Then the TPM of REMC is given by A = F +EG'

Where G' is defined as

$$G^{'} = (\sum_{h=0}^{Kv} \mathrm{C}_{\mathrm{k}})D \tag{3}$$

6. METHODOLOGY OF ANALSIS

The steps involved in the analysis of GSPN model are:

(1) Developing the reachability tree

(2) Constructing TPM of EMC and from that REMC of the GSPN model

(3) Determining steady state probabilities using the REMC

To ensure exisistance of unique steady state probabilities for the GSPN marking the following simplifying assumption are made.

- The GSPN is bounded. i.e. the reachability set is finite.
- Firing rates do not depend upon the on the time parameter
- The GSPN model is proper and deadlock free

Let Y be a vector number. Then solution to

$$YA = Y \text{ and}$$

$$\sum_{i=1}^{n} y_i = 1 \tag{4}$$

Gives the stationary probabilities of the REMC. This can be interpreted as the relative number of visits to M_i.

Let m_i be the mean sojourn time of the marking M_i and is given by

$$m_i = \frac{1}{\sum_{t_k \in T_i} F(M_i, t_k)} \tag{5}$$

Where Ti is the set of enabled transitions in M_i.

Let q_i be the stedy state probability of marking M_i then

$$q_i = \frac{y_i . m_i}{\sum_{j=0}^{t} y_j . m_j} \tag{6}$$

Once the steady state probabilities of the tangible states were calculated, different performance measures can be computed as given below.

1. Probabilities that a particular condition 'c' holds:

$$prob(C) = \sum_{j \in s_1} q_j \tag{7}$$

Where S_i is the set of marking in which the condition 'c' is satisfied.

2. Probabilities that a place P_I has exactly tokens:

$$\text{Prob } (P_i, K) = \sum_{j \in s_2} q_i \tag{8}$$

Where S_2 is the set of marking which have exactly K tokens in the place p_i

3. Expected number of tokens in a place:

$$\text{ET } (P_i) = \sum_{k=1}^{k} k \text{Prob } (P_i, k) \tag{9}$$

K is the maximum number of tokens P_i may contain.

4. Throughput rate of an timed transition t_j:

$$TR(t_j) = \sum q_i F(M_i, t_j) r_{ij} \tag{10}$$

Where S_3 is the set of marking in which t_j is enabled.

r_{ij} =1 if tj is not in conflict with any of enabled transitions in M_i. Otherwise r_{ij} is the probability that t_j fires among the conflicting transitions.

5. Mean waiting time in a place p_i :

$$\text{Wait } (P_i) = \frac{ET(P_i)}{\sum_{t_j \in IT(P_i)} TR(t_j)} \tag{11}$$

Where it (p_j) is set of input transitions of p_i.

7. Analysis of manufacturing system with deadlock

Figure 3(a) and (b) give the EMC and REMC of the system. The following sequence of events will lead to deadlock state. Let initially the AGV and the machine be free and raw parts are available.

Then (i) the AGV carries a raw part and loads it to the machine, (ii) the machine starts processing the part,(iii) in the meantime, AGV returns to L\U station and carries another raw part machine and waits there for the machine to become free,(iv) the machine finishes processing the part and waits for the AGV to remove and carry away the part. Thus the two resources, machine and AGV are involved in a deadlock since each keeps waiting for the other indefinitely. Even if buffer space is provided then also a deadlock can still occur since the AGV may fill the entire buffer while the machine is processing a part. Reachability graph can be effectively used to prevent deadlocks. The deadlock d1 can be prevented by firing t1 in preference to t2 in marking m1.this means that AGV is to be assigned to a raw part when there is no finished part waiting. The deadlock D2 can be prevented by firing t6 in preference to t5 in the marking M4, i.e. not releasing AGV after machine starts processing a raw part. This deadlock can also prevented by firing t2 in marking M7, by assigning AGV to a finished part when a finished part is waiting.

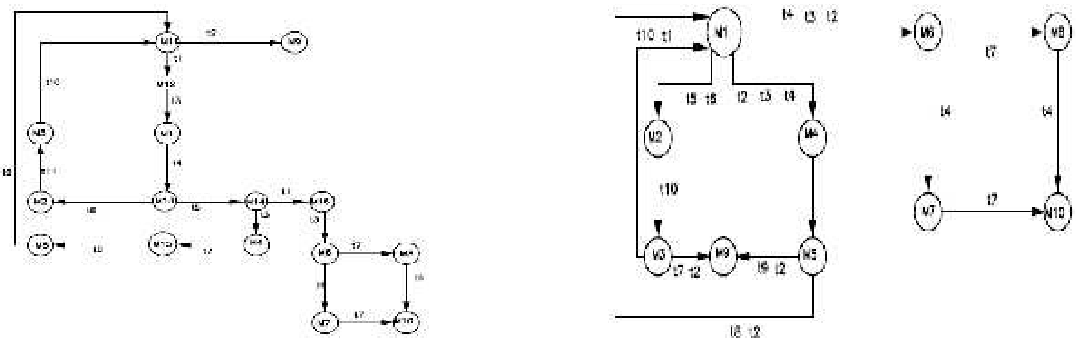
The marking processes of a GSPN having deadlock marking are equivalent to that of a Markov chain with absorbing states. The TPM of the REMC can be divided into four matrices with T being the matrix of transition probabilities among tangible marking gives the transition probabilities from tangible to deadlocked markings, an identify matrix and a zero element matrix. The fundamental matrix is defined as $F= (I-T)^{-1}$

F_{ij} gives the mean number of times tangible marking M_j is visited starting from M_i before reaching a deadlock. G = FC gives the long term probability that the marking process reaches the j th deadlock starting from marking M_i. Table 1 gives the performance measures. Interpretations of tangible marking are

M1 (2, 5, 7)	AGV carrying raw part to machine idle
M2 (2, 11)	machine processing part, AGV not released
M3 (2, 7, 12)	AGV, not released during processing by machine, carrying finished part Machine idle
M4 (2, 8)	machine processing part, AGV released
M5 (2, 7, 10)	AGV carrying finished part machine idle
M6 (2, 5, 8)	AGV carrying raw part to machine, machine processing a part
M7 (2, 6, 8)	AGV with raw part waiting for machine, machine processing a part
M1 (2, 5, 7)	AGV carrying raw part to machine, machine processing and waiting for AGV to carry (unload) the raw part

Deadlock sates

I	M9 (2 4 7)	machine idle, AGV available to carry a finished part
II	M10 (2 6 9)	AGV with raw part waiting for machine, machine with finished Part waiting for AGV



(a) (b)

Figure3(a) EMC of Manufacturing system with Deadlocks GSPN Model (b) REMC of Manufacturing system with Deadlocks GSPN Model

8. Results & Discussions

A modelling tool namely Generalized Stochastic Petri nets System (GSPN) is used for performance evaluation of manufacturing systems. Comparison of processing time and transformation time for the machine and AGV has been performed. Two cases are considered, the details of which are provided in Table 1.

Table 1. Results of GSPN Analysis of manufacturing system with Deadlocks

Processing and Transportation Times:	Case (i)	Case (ii)
Time taken by AGV to carry Raw Part to machine	8	6

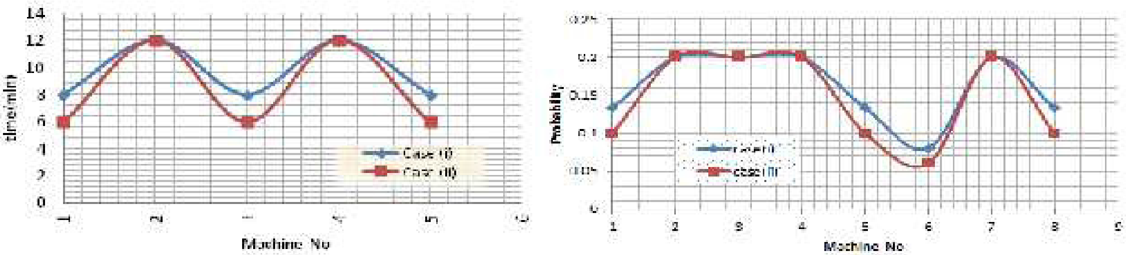
Processing time on Machine, AGV released	12	12
Time taken by AGV to Carry Finished part to L/UL station	8	6
Processing time on Machine, AGV not released	12	12
Time taken by AGV to Carry Finished part, AGV didn't release During processing of part	8	6

Table 2. Results of machine [1-8] Vs probability

	Case (i)	case (ii)
M (1)	0.1333	0.1000
M (2)	0.2000	0.2000
M (3)	0.2000	0.2000
M (4)	0.2000	0.2000
M (5)	0.1333	0.1000
M (6)	0.0800	0.0600
M (7)	0.2000	0.2000
M (8)	0.1333	0.1000

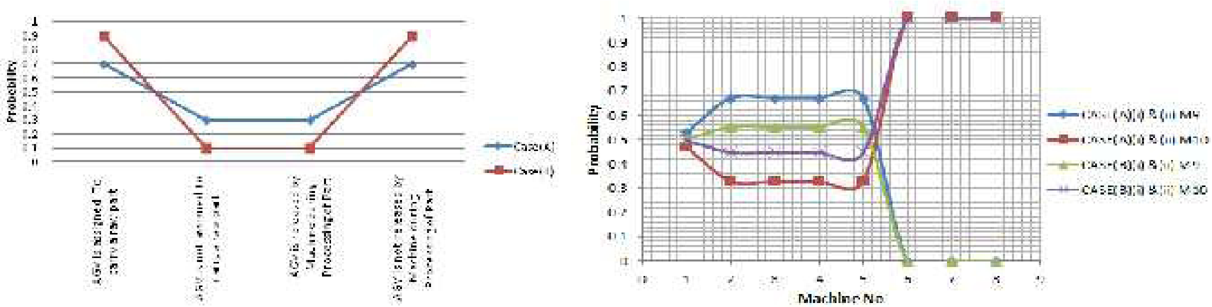
Table 3. Routing Probabilities

Routing Probabilities	Case (A)	Case (B)
Probability that AGV is assigned To carry a raw part	0.7	0.9
Probability that AGV is not assigned To carry a raw part	0.3	0.1
Probability that AGV is released by Machine during Processing of Part	0.3	0.1
Probability that AGV is not released by Machine during Processing of Part	0.7	0.9



(a) (b)
Figure 4 (a) Analysis of manufacturing system with Deadlocks (b) Results of Probability Vs. Machine no.

It can be seen from Figure 4(a) shows that the processing time of machine is same in case (i) and (ii) when AGV is released and not rereleased. In figure 4(b) machine 2, 4 and 7 maximum probability we get. Table 2. Shows the results of probability of machine [1-8]. This result is finding out by using Mat-Lab programming.



(a) (b)
Figure 5(a) shows that routing probability of manufacturing system with Deadlocks (b) Case (A) & (B) shows the probability Vs M/C no

Table 3.Model given Mean Time Taken before Reaching Deadlock

	Case (A)		Case (B)	
	Case (i)	Case(ii)	Case (i)	Case (ii)
Mean Time Taken Before Reaching Deadlock (Hrs.)	0.4628	1.02	2.87	2.67

Table 4. Probability that the System Moves From Tangible State 'i' To Deadlock State 'j' is

	CASE (A) (i) & (ii)		CASE (B) (i) & (ii)	
	M9	M10	M9	M10
M 1	0.5302	0.4698	0.5028	0.4972
M 2	0.6711	0.3289	0.5525	0.4475
M 3	0.6711	0.3289	0.5525	0.4475
M 4	0.6711	0.3289	0.5525	0.4475
M 5	0.6711	0.3289	0.5525	0.4475
M 6	0	1	0	1
M 7	0	1	0	1
M 8	0	1	0	1

In the Table 4 present the results of machine 1 to 8 in two case (A) and (B). case(A) is also the present the results of case(i) and (ii) for machine no 9 and 10. Fig. 6 helps in visualizing the different cases as discussed in Table 4. Probability are Case (A) &(B) shows the results of machine 6-8 linear. Probability is in both cases for machine is similar.

Conclusion

The developed GSPN model is analyzed using Markovian methods. From the reachability tree the transition probability matrix of the reduced embedded Markov chain is computed. Mat lab functions are used to get the routing probabilities and from them steady state probabilities are calculated various performance measures were computed for all the models developed. In This paper discussed a scheduling problems and the results pertaining to reachability tree analysis.GSPN provides an opportunity to visualize the manufacturing systems modeling in different aspect.

The following conclusions can be drawn

1. A substantial difference between the production time for the eight machines is observed
2. Machine 2,3,4 and 7 give maximum probability.

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